



Reason Abstractly and Quantitatively in Mathematics

Competency

Educators will engage and support students as they learn to reason abstractly and quantitatively.

Key Method

Educators will analyze a student task, and the student proficiency levels of the students and explain Mathematical Practice Standard 2, *Reasoning Abstractly and Quantitatively in Mathematics*, to stakeholders.

Method Components

Mathematical Practices

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important “processes and proficiencies” with longstanding importance in mathematics education.

The first of these is the [NCTM process standards from the National Council of Teachers of Mathematics \(NCTM\)](#). The second is the strands of mathematical proficiency specified in the National Research Council's report [Adding It Up](#).

These process standards are:

- Problem-Solving
- Reasoning and Proof



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- Communication
- Connections
- Representation

Strive to build these dispositions in students:

- Adaptive reasoning
- Strategic competence
- Conceptual understanding
- Procedural fluency
- Productive struggle

These are the eight standards for mathematical practice:

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of the structure
8. Look for and Express regularity in repeated reasoning

Talking about Mathematics: The importance of language in mathematics instruction and practice

Mathematics is a language and students need to become proficient in it. In the process, students need to understand and routinely apply the language of mathematics, which includes vocabulary, symbols, and sentence structures. The language of mathematics crosses all the standards for mathematical practice, so providing ample opportunities for students to learn and apply this language, both independently and collaboratively, will support their mathematical proficiency.

Explaining math can include the following:

- Words
- Pictures
- Mathematical expressions
- Using visual aids such as graphs
- Number lines
- Diagrams
- Drawing shapes with arm movement

Using the visuals above will help all students but especially those with multi-lingual skills.



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Assessing Mathematics

Assessing student learning is vital to progress in learning and steering instruction to ensure learning growth. It's important to ensure that assessments include these core components:

- Outcomes that are clear and measurable
- Opportunities for learners to show in a variety of ways, that they understand outcomes
- Assessment results that can be used to change instruction as needed to improve student learning

Abstract and Quantitative Reasoning

Reasoning abstractly and quantitatively requires learners to identify important quantities and relationships between those quantities in a problem situation, and then use mathematics (numbers, manipulatives, diagrams, graphs, algebraic expressions, etc.) to represent them and solve the problem. Abstract and quantitative reasoning asks students to move back and forth between the problem context and the mathematics used to represent it.

Proficiency in quantitative reasoning can:

- Create a coherent representation of the problem
- Consider the units involved
- Attend to the meaning of quantities and the relationships between them

Assessing Abstract and Quantitative Reasoning

Assessing student learning is vital to progress in learning and steering instruction to ensure learning growth. Quantitative Reasoning (QR) Assessments follow traditional assessment methods such as multiple choice questions to rubrics. Although it is important to ensure the assessment used for mathematics assesses the thinking process, the core components of assessments are the foundation:

Core components of assessment include:

- Outcomes that are clear and measurable
- Opportunities for learners to show that they understand outcomes in a variety of ways. Assessment results that can be used to change instruction as needed to improve student learning



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Teaching Strategies that Support Mathematical Reasoning

Pedagogical strategies that support mathematical thinkers are essential to use. Celebrate the use of models and different types of math reasoning.

- Ask non-routine problems
- Encourage students to use multiple representations of their mathematical thinking and reasoning
- Encourage students to use visual representations and manipulatives to solve problems and in their reflection
- Ask learners to find multiple solutions and to pursue multiple solution strategies.
- Have learners share understandings in various ways
- Use talk moves
- Actively listen
- Celebrate multiple ways students solve mathematical problems
- Use read-alouds to spark discussion
- Praise perseverance and growth

Supporting Rationale and Research

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Keazer, L., & Jung, H. (2020). Prospective teachers anticipate challenges fostering the mathematical practice of making sense. *School Science and Mathematics*, 120(2), 78-89. <https://onlinelibrary.wiley.com/doi/10.1111/ssm.12390>

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Rosenfeld, R. (2020, October 4). *Extending the Mathematical Practice Standards Across the Curriculum*. WestEd. Retrieved November 7, 2022, from

<https://www.wested.org/wested-bulletin/insights-impact/mathematical-practice-standards-across-the-curriculum/>

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The University of Texas at Austin Charles A. Dana Center. (2022). *Mathematical Practice Standards*. Inside Mathematics. Retrieved November 7, 2022, from

<https://www.insidemathematics.org/common-core-resources/mathematical-practice-standards>

Resources

Curriculum and Standards

[Classroom Resources - National Council of Teachers of Mathematics](#)

[Achievethecore.org: Instructional Content Nav - Mathematics: Focus by Grade Level](#)



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[Deep Dive: How Math Practices 1–3 Help All Students Access Math Learning and Build Skills for the Future](#)

[Math Ed Reports](#)

[Mathematics Standards](#)

[MP2 Examples –Standard 2: Reason Abstractly & Quantitatively](#)

[MP2 Reason abstractly and quantitatively](#)

[Process - National Council of Teachers of Mathematics](#)

[Standard 2: Reason Abstractly & Quantitatively | Inside Mathematics](#)

[Standards for Mathematical Practice](#)

[The Standards for Mathematical Practice & Instructional Materials - Peers and Pedagogy](#)

Strategies for Mathematics

[6 Simple \(But Effective\) Instructional Strategies for Mathematics](#)

[9 Easy Tips for Teaching Standards for Mathematical Practice](#)

[Assessing Quantitative Reasoning](#)

[Math Reasoning: Problem-Solving Strategies](#)

[Talk moves](#)

[Thinking Mathematically](#)

[When does \$2 + 7 + 8 = 1\$? NCTM Ignite Talk](#)

[FosteringMATHPractices](#) Capturing Quantities

Submission Guidelines & Evaluation Criteria



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To earn the micro-credential, you must receive a passing score in Parts 1 and 3 and be proficient in all components in Part 2.

Part 1. Overview Questions (Provides Context)

(250-400)

Please use the suggested word count as a guide to answer the following contextual questions. This will help our assessor understand your current context for working on this micro-credential.

Please do not include any information that will make you identifiable to your reviewers.

1. Why did you choose to work on this micro-credential? (Include your own career goals and your current teaching assignment.)
2. What is your experience with Math Practices?
3. Describe your student demographics.
4. What would you like your students to be able to do as a result of working on this micro-credential?

Passing: Response provides reasonable and accurate information that justifies the reason for choosing this micro-credential to address the specific needs of both the teacher and the student. Clearly state a learning goal that describes what you hope to gain from earning this micro-credential and how your students may benefit.

Part 2. Work Examples/Artifacts/Evidence

To earn this micro-credential, please submit the following three artifacts as evidence of your learning.

**Please do not include any information that will make you or your students identifiable to your reviewers.*

Artifact 1: Task Analysis

Choose a problem or task from a lesson that allows students to engage in Math Practice 2. Utilize the implementation characteristics identified under Math Practice 2 on [Achieve the Core's Implementing Standards for Mathematical Practices](#) to analyze the problem or task.

Task analysis must include:

- The problem or task



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- The mathematics standard(s) addressed and any learning objectives/outcomes
- How the task addresses the implementation characteristics (in the bottom box)

Artifact 2: Student evidence from task

Teach the task you analyzed in Artifact 1 and upload two student work samples that reflect different levels of proficiency of Math Practice 2. The samples must include:

- Annotations that identify at least one strength for each student
- Annotations that identify at least one step toward proficiency for each student

OR

Make a two- to four-minute video that shows you teaching and interacting with students for the task you analyzed in Artifact 1

Your video must include:

- A demonstration of how you verbally identify one strength (either during the recorded conversation or in a voiceover)
 - For example: While working with your student(s), pause and say something like, "I am noticing that you are good at _____."
- A demonstration of how you verbally identify at least one step toward proficiency for each student (either during the recorded conversation or in a voiceover)
 - For example: At the end of the lesson, say to your student(s) "I am so proud of the way you----. I think a good next step for you to work on is----."

Follow your district's mandated permissions for photography with students.

Video tips: Video tape students from the back of the head with instruments blocking students' faces and/or video from the neck down. Narrate or display information to explain the learning intended for the Math Practice.

**You will need to upload your video to YouTube or another video hosting platform. Please make sure your sharing permissions are set to anyone with the link can view it. You can change this after you have earned this micro-credential*

Artifact 3: One Page Handout

Use [Achieve the Core's Implementing Standards for Mathematical Practices](#) to create a one-page handout for families and/or support professionals that synthesizes how they can support students at home or at school, to reason abstractly and quantitatively. Your one-page handout needs to include:



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- Your intended audience: families, support professionals, or both
- Explanation of Math Practice 2.
- What students should be able to do (in a language students can understand)
- 2-3 questions and/or prompts to guide students through the process
- Instructions on what to do when a student is stuck
- 1-3 Resources/links to use at home, in a resource room, or another location

Part 2. Rubric

	Proficient	Basic	Developing
Artifact 1: Task Analysis	<p>The task analysis includes all the following:</p> <ul style="list-style-type: none"> • the problem and task • The math standard(s) and objectives & outcomes • The Four Implementation Characteristics of Mathematical Practice 2 • How the task addresses the implementation characteristics 	<p>The task analysis includes some of the following:</p> <ul style="list-style-type: none"> • The problem and task • The math standard(s) and objectives & outcomes • The Four Implementation Characteristics of Mathematical Practice 2 • How the task addresses the implementation characteristics 	<p>The task analysis does not sufficiently include all the following:</p> <ul style="list-style-type: none"> • The problem and task • The math standard(s) and objectives & outcomes • The Four Implementation Characteristics of Mathematical Practice 2 • How the task addresses the implementation characteristics
Artifact 2: Student Evidence from Task Samples of Student Work Option	<p>Student samples include all the following:</p> <ul style="list-style-type: none"> • Two samples that display different levels of 	<p>Student samples include some of the following:</p> <ul style="list-style-type: none"> • Two samples that display different levels of 	<p>Student samples do not sufficiently include all of:</p> <ul style="list-style-type: none"> • Two samples that display different levels of



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	<p>Math Practice 2</p> <ul style="list-style-type: none"> • Written annotations that identify at least one strength for each student • Written annotations that identify at least one step toward proficiency for each student 	<p>Math Practice 2</p> <ul style="list-style-type: none"> • Written annotations that identify at least one strength for each student • written annotations that identify at least one step toward proficiency for each student 	<p>Math Practice 2</p> <ul style="list-style-type: none"> • Written annotations that identify at least one strength for each student • written annotations that identify at least one step toward proficiency for each student
<p>Artifact 2: Student evidence from task</p> <p>Video Option</p>	<p>The video includes all the following:</p> <ul style="list-style-type: none"> • At least one strength (either during the recorded conversation or in a voiceover) • At least one step toward proficiency for each student (either during the recorded conversation or in a voiceover) 	<p>Videos include some of the following:</p> <ul style="list-style-type: none"> • At least one strength (either during the recorded conversation or in a voiceover) • At least one step toward proficiency for each student (either during the recorded conversation or in a voiceover) 	<p>Videos do not sufficiently include these:</p> <ul style="list-style-type: none"> • At least one strength (either during the recorded conversation or in a voiceover) • At least one step toward proficiency for each student (either during the recorded conversation or in a voiceover)
<p>Artifact 3: One-Page Handout</p>	<p>The one-page handout shows how to help</p>	<p>The one-page handout attempts to show how to</p>	<p>The one-page handout does not show how to help</p>



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	<p>students reason abstractly and quantitatively.</p> <p>And includes all the following:</p> <ul style="list-style-type: none"> • Your intended audience: families, support professionals, or both • Explanation of Math Practice 2 • What students should be able to do (in language students will understand) • 2-3 questions and/or prompts to guide students through the process • What to do when a student is stuck • 1-3 resources/links to use at home, in a resource room, or in another location 	<p>help students reason abstractly and quantitatively</p> <p>It includes all but two of the following:</p> <ul style="list-style-type: none"> • Your intended audience: families, support professionals, or both • Explanation of Math Practice 2 • What students should be able to do (in language students will understand) • 2-3 questions and/or prompts to guide students through the process • What to do when a student is stuck • 1-3 resources/links to use at home, in a resource room, or in another location 	<p>students reason abstractly and quantitatively.</p> <p>It is missing three or more of the below::</p> <ul style="list-style-type: none"> • Your intended audience: families, support professionals, or both • Explanation of Math Practice 2 • What students should be able to do (in language students will understand) • 2-3 questions and/or prompts to guide students through the process • What to do when a student is stuck • 1-3 resources/links to use at home, in a resource room, or in another location
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Part 3 Reflection

(350-600 words)

Use the word count as a guide to write a personal reflection about your work on this micro-credential. For tips on writing a good reflection, review this resource:

[How Do I Write a Good Personal Reflection?](#)

Please do not include any information that will make you identifiable to your reviewers.

1. How did thinking about Math Practice 2 impact how you approached this task? Describe any shifts in understanding for yourself and your students.
2. Reflect on your role in facilitating student learning regarding Math Practice 2 on this task. In your response, include specific details from student work and your feedback on Artifact 2.
3. What next steps might you take moving forward? How do you plan to make this process part of your regular practice?

Passing: Reflection provides evidence that this activity has positively impacted both educator practice and student success. Specific examples are cited directly from personal or work-related experiences to support claims. Also included are specific, actionable steps that demonstrate how new learning will be integrated into future practices.



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