



# Look for and Express Regularity in Repeated Reasoning

## Competency

Educators will teach students to look for and express regularity in repeated reasoning.

## Key Method

Educators will analyze a student task and proficiency levels to explain Mathematical Practices Standard 8, Look for and Express Regularity in Repeated Reasoning, to stakeholders.

## Method Components

### Mathematical Practices

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important “processes and proficiencies” with longstanding importance in mathematics education.

The first of these is the [NCTM process standards from the National Council of Teachers of Mathematics \(NCTM\)](#). The second is the strands of mathematical proficiency specified in the National Research Council's report [Adding It Up](#).

These process standards are:

- Problem-Solving
- Reasoning and Proof
- Communication



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- Connections
- Representation

Strive to build these dispositions in students:

- Adaptive reasoning
- Strategic competence
- Conceptual understanding
- Procedural fluency
- Productive struggle

These are the eight standards for mathematical practice:

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and Express regularity in repeated reasoning

## Talking about Mathematics: The importance of language in mathematics instruction and practice

Mathematics is a language and students need to become proficient in it. In the process, students need to understand and routinely apply the language of mathematics, which includes vocabulary, symbols, and sentence structures. The language of mathematics crosses all the standards for mathematical practice, so providing ample opportunities for students to learn and apply this language, both independently and collaboratively, will support their mathematical proficiency.

Explaining math can include the following:

- Words
- Pictures
- Mathematical expressions
- Using visual aids such as graphs
- Number lines
- Diagrams
- Drawing shapes with arm movement

Using the visuals above will help all students but especially those with multi-lingual skills.

## Assessing Mathematics



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Assessing student learning is vital to progress in learning and steering instruction to ensure learning growth. It's important to ensure that assessments include these core components:

- Outcomes that are clear and measurable
- Opportunities for learners to show in a variety of ways, that they understand outcomes
- Assessment results that can be used to change instruction as needed to improve student learning

## Looking for and Expressing Regularity in Repeated Reasoning

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again and conclude they have a repeating decimal.

By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation  $(y - 2)/(x - 1) = 3$ . Noticing the regularity in the way terms cancel when expanding  $(x - 1)(x + 1)$ ,  $(x - 1)(x^2 + x + 1)$ , and  $(x - 1)(x^3 + x^2 + x + 1)$  might lead them to the general formula for the sum of a geometric series.

As they work to solve a problem, mathematically proficient students maintain oversight of the process while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Mathematically proficient students should be able to:

- Notice if calculations are repeated, and look both for general methods and for shortcuts.
- Maintain oversight of the process, while attending to the details.
- Continually evaluate the reasonableness of their intermediate results.

## Teaching Strategies that Support Repeated Reasoning

Pedagogical strategies that support repeated reasoning are important when planning math instruction. Here are a couple of sample strategies to prompt and develop repeated reasoning:

- Engage in a mathematical process (i.e., counting, calculating, and constructing) and sense regularity in the way you are counting, calculating, and constructing.
- Ask yourself, "Am I counting/calculating/constructing in the same way each time?" and "What about this process is repeating?"



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# Supporting Rationale and Research

Clayton, H. (2014). Thinking Behind the Content: Standards for Mathematical Practice. Just ASK Publications. Retrieved November 7, 2022, from <https://justaskpublications.com/just-ask-resource-center/e-newsletters/msca/thinking-behind-the-content-standards-for-mathematical-practice/>

Elementary Math at EDC. (2020). *MP: Why do we need standards for Mathematical Practice? – Elementary Math*. Elementary Math. Retrieved November 7, 2022, from <https://elementarymath.edc.org/resources/why-do-we-need-standards-for-mathematical-practice/>

Ernst-Slavit, G., & Slavit, D. (2021). *Mathematically Speaking*. Language Magazine. Retrieved November 7, 2022, from <https://www.languagemagazine.com/mathematically-speaking/>

*Illustrative Mathematics*. (2016). Illustrative Mathematics. Retrieved November 7, 2022, from <http://tasks.illustrativemathematics.org/practice-standards/>

Jamison, R. E. (2000, May). Learning the Language of Mathematics. *Language and Learning Across the Disciplines*, 4(1), 45-54. <https://wac.colostate.edu/docs/llad/v4n1/jamison.pdf>

Keazer, L., & Jung, H. (2020). Prospective teachers anticipate challenges fostering the mathematical practice of making sense. *School Science and Mathematics*, 120(2), 78-89. <https://onlinelibrary.wiley.com/doi/10.1111/ssm.12390>

Made4Math. (2020, May 12). *9 Easy Tips for Teaching Standards for Mathematical Practice*. Made4Math. Retrieved November 7, 2022, from <https://www.made4math.com/standards-for-mathematical-practice/>

Rosenfeld, R. (2020, October 4). *Extending the Mathematical Practice Standards Across the Curriculum*. WestEd. Retrieved November 7, 2022, from <https://www.wested.org/wested-bulletin/insights-impact/mathematical-practice-standards-across-the-curriculum/>

The University of Texas at Austin Charles A. Dana Center. (2022). *Mathematical Practice Standards*. Inside Mathematics. Retrieved November 7, 2022, from <https://www.insidemathematics.org/common-core-resources/mathematical-practice-standards>

## Resources



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## Curriculum and Standards

[Classroom Resources - National Council of Teachers of Mathematics](#)

[Achievethecore.org : Instructional Content Nav - Mathematics: Focus by Grade Level](#)

[Deep Dive: How Math Practices 7 and 8 Power Student “Lightbulb” Moments](#)

[Math Ed Reports](#)

[Mathematics Standards](#)

[MP8 Examples](#)

[MP8 Look for and express regularity in repeated reasoning – Elementary Math](#)

[Process - National Council of Teachers of Mathematics](#)

[Standard 8: Look for & Express Regularity in Repeated Reasoning | Inside Mathematics](#)

[Standards for Mathematical Practice](#)

[The Standards for Mathematical Practice & Instructional Materials - Peers and Pedagogy](#)

## Assessment and Strategies for Mathematics

[6 Simple \(But Effective\) Instructional Strategies for Mathematics](#)

[9 Easy Tips for Teaching Standards for Mathematical Practice](#)

[Assessing Quantitative Reasoning](#)

[Math Reasoning: Problem Solving Strategies](#)

[Talk moves](#)

[When does  \$2 + 7 + 8 = 1\$ ? NCTM Ignite Talk](#)

[Fostering Math Practices Recognizing Repetition](#)

## Submission Guidelines & Evaluation Criteria



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To earn the micro-credential, you must receive a passing score in Parts 1 and 3 and be proficient for all components in Part 2.

## Part 1. Overview Questions (Provides Context)

(250-400)

Please use the suggested word count as a guide to answer the following contextual questions. This will help our assessor understand your current context for working on this micro-credential.

Please do not include any information that will make you identifiable to your reviewers.

1. Why did you choose to work on this micro-credential? (Include your own career goals and your current teaching assignment.)
2. What is your experience with Math Practices?
3. Describe your student demographics.
4. As a result of working on this micro-credential, what would you like your students to be able to?

**Passing:** Response provides reasonable and accurate information that justifies the reason for choosing this micro-credential to address the specific needs of both the teacher and the student. Clearly state a learning goal that describes what you hope to gain from earning this micro-credential and how your students may benefit.

## Part 2. Work Examples/Artifacts/Evidence

To earn this micro-credential please submit the following three artifacts as evidence of your learning.

Please do not include any information that will make you or your students identifiable to your reviewers.

### Artifact 1: Task Analysis

Extract a problem or task from a lesson that allows students to engage in Math Practice 8. Utilize the implementation characteristics identified under Math Practice 8 on [Achieve the Core's Implementing Standards for Mathematical Practices](#) to analyze the problem or task.

Task analysis must include the following:

- The problem or task



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- The mathematics standard(s) addressed and any learning objectives/outcomes
- How the task addresses the implementation characteristics

### **Artifact 2: Annotated Student work from task**

Teach the task you analyzed in Artifact 1 and upload two student work samples that reflect different levels of proficiency of Math Practice 8. The samples must include the following:

- Annotations that identify at least one strength for each student
- Annotations that identify at least one step toward proficiency for each student

### **OR**

Make a 2-4-minute video that shows you teaching and interacting with students for the task you analyzed in Artifact 1

Your video must include the following:

- Verbal identification of one strength (either during the recorded conversation or in a voiceover)
  - For example: While working with your student(s) pause and say something like, "I am noticing that you are really good at \_\_\_\_\_."
- Verbal identification of at least one step toward proficiency for each student (either during the recorded conversation or in a voiceover)
  - For example: At the end of the lesson say to your student(s) "I am so proud of the way you----. I think a good next step for you to work on is----."

***Follow your district's mandated permissions for photography with students.***

**Video tips:** Video students from the back of the head or from the neck down; do not include their faces. Narrate or display information to explain the learning intended for the Math Practice.

*You will need to upload your video to YouTube or another video hosting platform. Please make sure your sharing permissions are set to anyone with the link can view it. You can change this after you have earned this micro-credential.*

### **Artifact 3: One-Page Handout**

Use [Achieve the Core's Implementing Standards for Mathematical Practices](#) to create a one-page handout for families and/or support professionals that summarizes how they can support students at home or at school, to look for and make sense of repeated reasoning. Your one-page handout needs to include the following:

- Your intended audience: families, support professionals, or both
- Explanation of Math Practice 8



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- What students should be able to do (in student language)
- Questions and/or (2-3) prompts to guide students through the process
- What to do when a student is stuck
- Resources/links (1-3) to use at home, in a resource room, or another location

## Part 2. Rubric

	<b>Proficient</b>	<b>Basic</b>	<b>Developing</b>
<b>Artifact 1:</b> Task Analysis	<p>The task analysis includes all the following:</p> <ul style="list-style-type: none"> <li>• The problem and task</li> <li>• The math standard(s) and objectives and outcomes</li> <li>• The four Implementation Characteristics of Mathematical Practice 8</li> <li>• how the task addresses the implementation characteristics</li> </ul>	<p>The task analysis includes some of the following:</p> <ul style="list-style-type: none"> <li>• The problem and task</li> <li>• The math standard(s) and objectives and outcomes</li> <li>• The four Implementation Characteristics of Mathematical Practice 8</li> <li>• How the task addresses the implementation characteristics</li> </ul>	<p>The task analysis does not sufficiently include the following:</p> <ul style="list-style-type: none"> <li>• The problem and task</li> <li>• The math standard(s) and objectives and outcomes</li> <li>• The four Implementation Characteristics of Mathematical Practice 8</li> <li>• How the task addresses the implementation characteristics</li> </ul>
<b>Artifact 2:</b> Student Evidence from Task  Samples of Student Work <b>Option</b>	<p>Student samples include all the following:</p> <ul style="list-style-type: none"> <li>• Two samples that display different levels of Math Practice 8</li> <li>• Written annotations that identify at least one strength for each student</li> </ul>	<p>Student samples include some of the following:</p> <ul style="list-style-type: none"> <li>• two samples that display different levels of Math Practice 8</li> <li>• Written annotations that identify at least one strength for each student</li> <li>• Written annotations that</li> </ul>	<p>Student samples do not include the following:</p> <ul style="list-style-type: none"> <li>• Two samples that display different levels of Math Practice 8</li> <li>• Written annotations that identify at least one strength for each student</li> <li>• Written annotations that identify at least</li> </ul>



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	<ul style="list-style-type: none"> <li>Written annotations that identify at least one step toward proficiency for each student</li> </ul>	<p>identify at least one step toward proficiency for each student</p>	<p>one step toward proficiency for each student</p>
<p><b>Artifact 2:</b> Student Evidence from Task</p> <p>Video Option</p>	<p>Video includes all the following:</p> <ul style="list-style-type: none"> <li>At least one strength (either during the recorded conversation or in a voiceover)</li> <li>At least one step toward proficiency for each student (during the recorded conversation or in a voiceover)</li> </ul>	<p>Videos include some of the following:</p> <ul style="list-style-type: none"> <li>At least one strength (either during the recorded conversation or in a voiceover)</li> <li>At least one step toward proficiency for each student (during the recorded conversation or in a voiceover)</li> </ul>	<p>Videos do not sufficiently include the following:</p> <ul style="list-style-type: none"> <li>At least one strength (either during the recorded conversation or in a voiceover)</li> <li>At least one step toward proficiency for each student (during the recorded conversation or in a voiceover)</li> </ul>
<p><b>Artifact 3:</b> One Page Handout</p>	<p>The one-page handout clearly shows how to help students solve problems and persevere through them.</p> <p>And includes all the following:</p> <ul style="list-style-type: none"> <li>Your intended audience: families, support professionals, or both</li> <li>Explanation of Math Practice 8</li> <li>What students should be able to do (in</li> </ul>	<p>The one-page handout attempts to show how to support students to increase the achievement of making sense of solving problems and persevere through them.</p> <p>It includes all but two of the following:</p> <ul style="list-style-type: none"> <li>Your intended audience: families, support professionals, or both</li> <li>Explanation of Math Practice 8</li> </ul>	<p>The one-page handout does not show how to support students to increase the achievement of making sense of solving problems and persevere through them.</p> <p>It is missing three or more of the following:</p> <ul style="list-style-type: none"> <li>Your intended audience: families, support professionals, or both</li> <li>Explanation of Math Practice 8</li> </ul>



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	<p>language students can understand )</p> <ul style="list-style-type: none"> <li>• Questions and/or prompts (2-3) to guide students through the process</li> <li>• What to do when a student is stuck</li> <li>• Resources/links (1-3) to use at home, in a resource room, or in another location</li> </ul>	<ul style="list-style-type: none"> <li>• What students should be able to do (in language students can understand )</li> <li>• Questions and/or prompts (2-3) to guide students through the process</li> <li>• What to do when a student is stuck</li> <li>• Resources/links (1-3) to use at home, in a resource room, or in another location</li> </ul>	<ul style="list-style-type: none"> <li>• What students should be able to do (in language students can understand)</li> <li>• Questions and/or prompts (2-3) to guide students through the process</li> <li>• What to do when a student is stuck</li> <li>• Resources/links (1-3) to use at home, in a resource room, or in another location</li> </ul>
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### Part 3 Reflection

(400-600 words)

Use the word count as a guide to write a personal reflection about your work on this micro-credential. For tips on writing a good reflection, review this resource:

[How Do I Write a Good Personal Reflection?](#)

*Please do not include any information that will make you identifiable to your reviewers.*

1. How did thinking about Math Practice 8 impact how you approached this task? Describe any shifts in understanding for yourself and for your students.
2. Reflect on your role in facilitating student learning regarding Math Practice 8 on this task. In your response, include specific details from student work and your feedback on Artifact 2.
3. What next steps can you take moving forward? How do you plan to make this process part of your regular practice?

**Passing:** Reflection provides evidence that this activity has had a positive impact on both educator practice and student success. Specific examples are cited from



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personal or work-related experiences to support claims. Also included are specific actionable steps that demonstrate how new learning will be integrated into future practices.



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